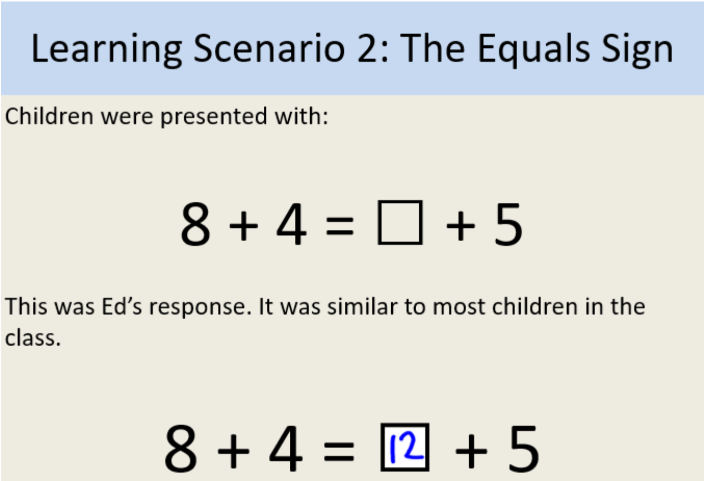
Learning Scenario 2



In this Learning Scenario, Ed demonstrates a partial understanding, as he knows that 8 + 4 = 12. However, he has misunderstood the significance of the equal sign as he has mistakenly identified it as the place where the answer is shown in a calculation. Rather than understanding that it is representing equivalence (Haylock, 2017) with the following addition calculation. This misconception may have arisen in Year 1 where students are taught to interpret mathematical calculations involving addition, subtraction and equals signs (DfE, 2014). Within this unit of study, Ed may have only seen the equal sign at the end of a calculation and therefore misinterpreted its meaning. This misconception would need to be challenged before the teaching of greater than, less than and equal signs in Year 2 (DfE, 2014). As misconceptions need to be addressed in a meaningful context to facilitate higher level mathematics understanding (Hansen, 2014).

Ed's response was similar to most children in the class, suggesting that the class has had limited discrete teaching of equivalence, resulting in a misconception. Cockburn (2008) suggests that misconceptions expose children's partial understanding which allows teachers to identify and challenge them. To support teachers, the National Centre for Excellence in the Teaching of Mathematics have produced guidance to underpin the National Curriculum. The guidance introduces missing numbers problems into Year one as well as the discrete teaching of '= <>' (2014). This teaching alongside children's prior understanding of number bonds, known number facts and the inverse of addition (Hansen, 2008: Haylock, 2014) will support children in their understanding of missing number problems. Therefore, to address this misconception, I will build upon the children's foundation knowledge through a whole class activity.

Before reviewing the missing number problem, I would engage the class in discrete teaching of '= <>'. I would introduce this through the picture book, 'Please Mr Panda' by Steve Antony. I have chosen a high-quality text, as children's experiences in mathematics need to be engaging (Cockburn, 2008: Fox, 2001) and the familiar context supports learning. This book is about sharing doughnuts therefore, I have created felt doughnuts as a supporting enactive resource (Bruner, 1996). I have chosen to use enactive resources because they have a "strong visual and tactile appeal that relates well to how children learn" (Delaney, 2001 p.125). This active learning (Delaney, 2001) supports the inverse of addition, which underpins missing number problems (Hansen, 2008). Therefore, I would start by modelling two characters having different quantities of doughnuts and asking the children, 'who has the greater number of doughnuts?' as shown in figure 1. This introduces mathematical language, as well as the concepts of '<> =' Which is seen in the Year 2 National Curriculum, as they learn to compare and order numbers up to 100. Children then have the opportunity to explain their thinking, developing their mathematical understanding (Hansen, 2014) before working independently.

Figure 1

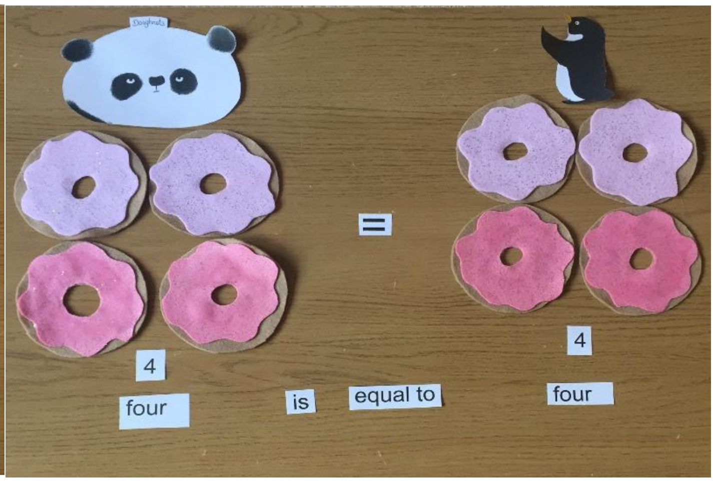


Figure 2

This also provides the opportunity to discretely teach equivalence, as seen in figure 2. When resources are combined for modelling and exploration by children, their learning is accelerated (Delaney, 2001). Therefore, in this activity the teacher can then observe each mixed ability pair physically model '<>=' and mathematically justify their reasoning to their partner, which will build upon their fluency which is an aim of the National Curriculum. This could be further deepened through higher-level questioning from the teacher, facilitating essential discussion and therefore reasoning (Hansen,2008). This will enable the teacher to assess their learning and when children have a concrete understanding of equivalence (Haylock, 2017), you could move onto missing number problems with enactive resources. I would continue to use the characters from 'Please Mr panda' and the doughnuts, however you could use counters or bears to represent this. I would not use a number line because they underpin comparison and missing number problems should be calculated using the inverse of addition (Hansen, 2008: Haylock, 2014).

Hansen (2008) further states that when teaching missing number problems, you should first introduce the missing number as the second number of the calculation for example 8 +□ = 32 rather than □ + 8 = 32, as children the latter more difficult. Thus, I would introduce missing number problems such as 8 + □= 20 (figure 3). Because the children can use their number bonds to 20 to support their calculations as well as writing and reading numbers to 20, which is a statuary requirement in Year 1 (DfE, 2014). Building on this success, I would then adapt the calculations to become 8 + 2 = 7 + □ as this is the calculation they originally struggled with. This will further develop their understanding of = being equivalence and therefore both sides needing to be equal, as shown in figure 4.

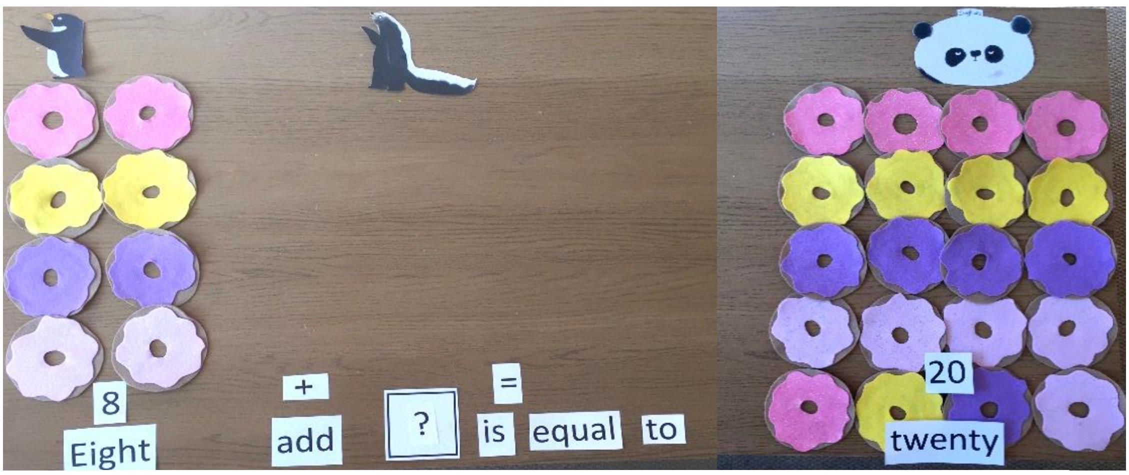


Figure 3

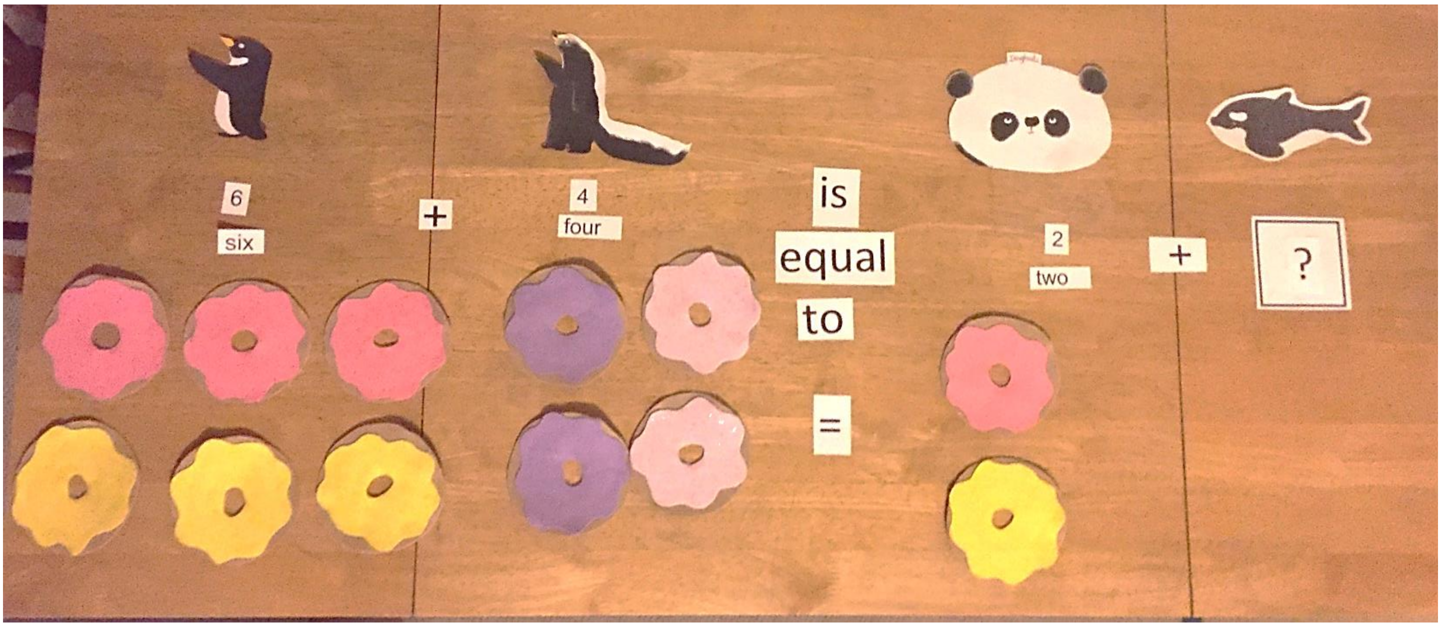


Figure 4

Through active discussion all children engage in the decision-making process through justification (Barrett, 2012) which underpins mathematical reasoning and therefore fluency (DfE, 2014). In addition, this activity could be described as Low Threshold, High Celling (LTHC), as all children will be able to access the initial activity and then extend learning by increasing the total up to 100 or rearranging the position of the digits within the calculation (McClure, 2011).

As a progression activity I would then keep the children in mixed ability pairs, so they can scaffold (Vygotsky, 1978) each other through a game. In the game the children are given calculations that they need to complete with differentiated challenges, illustrated in figure 5. The children then have to make the calculation balance; however, the complexity of their work is rewarded with points, the first to ten points wins. This naturally progresses from the previous enactive activity into a symbolic activity, however some children may wish to draw images underneath their work as a form of iconic representation (Bruner, 1996). This game provides opportunities for strategic thinking (Dillon, 2012) and differentiation scaffolds learners to be able to justify their reasoning with mathematical language (DFE, 2014) enabling them to problem solve effectively.

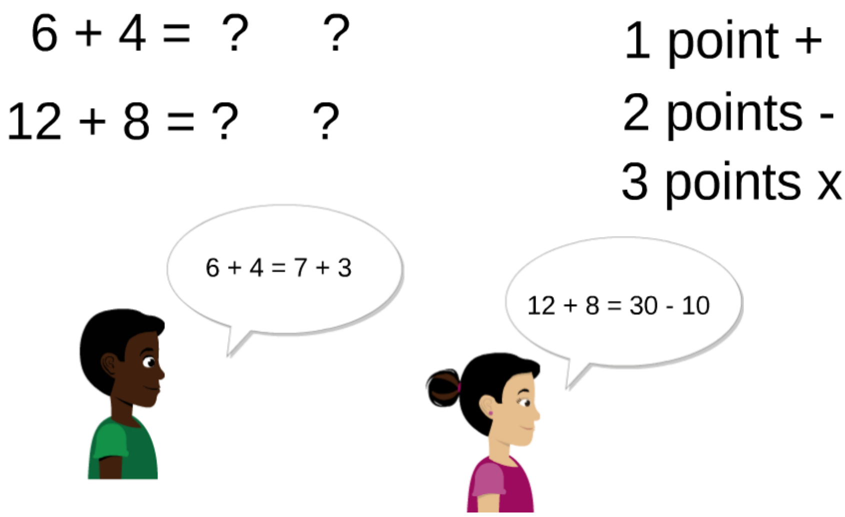
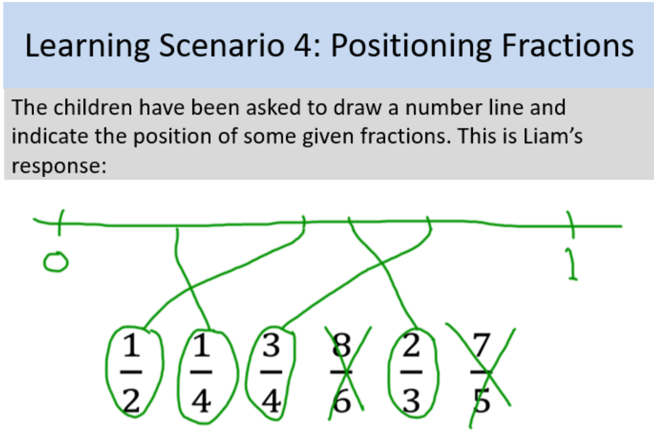


Figure 5

After this teaching, I would then return to the original missing number problem, as children will have relational understanding (Skemp, 1976) of equivalence through active learning and therefore can apply their new knowledge to the problem. By returning to the original question later, children have had the opportunity to complete the cycle of dialogue, which includes “articulation, re-formulation, reflection and resolution” (Ryan, 2007, p.31). Enabling children to obtain higher learning as they have explained their mathematical reasoning to their talk partner, underpinning their mathematical language. Children can then check to see if they have the correct answer using the inverse calculation which they will build upon in Year 3 (DfE, 2014). This also provides an opportunity for summative assessment (Monaghan, 2010) which allows the teacher to identify the progression of all children and assess the next steps in the learning journey.

Learning scenario 4



In this Scenario Liam has correctly positioned proper fractions on a number line, however he failed to order the improper fractions and instead has crossed them out, rather than understanding that they are greater than one. This could have occurred because Liam has only seen fractions as less than a whole, in combination with the misconception that improper fractions don't exists as numbers. Therefore, he has failed to meet the Year 5 aim to identify improper fractions and convert them into mixed numbers (DfE, 2014) as well as the Year 6 Curriculum aim to order and compare fractions >1 (DfE, 2014). Because Liam had correctly ordered the proper fractions on the number line, he is confident with his proper fraction knowledge and may have used his decimal and percentage knowledge to support this (DfE, 2014). However, he may have had limited experiences with improper fractions and mixed numbers.

To address the misconception, I would start by questioning Liam to unpick why he has crossed out the improper fractions. Then to develop Liam's understanding of improper fractions I would start by counting in familiar fractions using numicon, as demonstrated in figure 6. The denominator is illustrated through the bottom piece of numicon which underpins that the denominator is the bottom number (Haylock, 2014), whilst pegs illustrate the numerator on top, which will support Liam's mathematical language (DfE, 2014). This activity underpins the non-statutory guidance that pupils should practise counting in simple fractions (DfE, 2014). Counting in fractions also provides opportunities to develop fluency which is a further aim of the National Curriculum (DfE, 2014). This enactive resource (Bruner, 1966) also provides meaningful opportunities to reason mathematically with peers, whilst manipulating the resources which underpins children's understanding (Fosnot and Dolk, 2002). Whilst providing opportunities for teachers to assess in learning. (Monaghan, 2010).

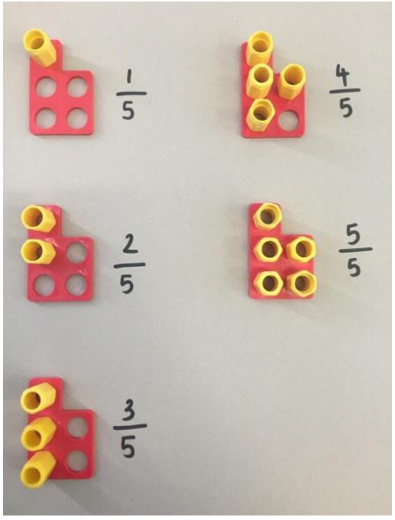


Figure 6

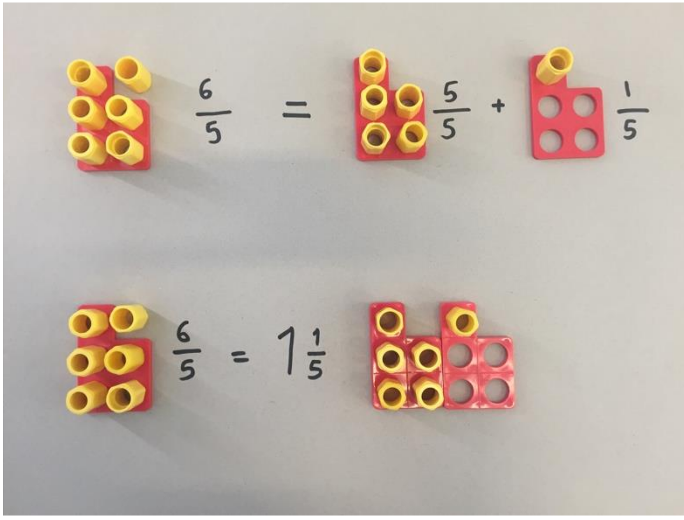


Figure 7

To further develop this learning, I will ask Liam critical questions (Hansen, 2014) such as, is 6/5 a proper fraction? Can we have fractions that are not proper? How might we represent this? These questions will require mathematical reasoning, supporting the aims of the National Curriculum is (DfE, 2014) whilst providing an insight into Liam's thinking (Hansen, 2017). I would first show Liam an improper fraction using the numicon as an enactive resource and then further model to Liam that improper fractions can be converted into mixed numbers, as demonstrated in figure 7. Explicit modelling of this is important because children often believe that fractions can only be less than 1 (Hansen, 2017) and Liam has clearly had limited experiences of fractions being greater than 1. This style of enactive modelling underpins fractions as part of a whole, and fractions as a number (Haylock, 2014) whilst providing further aural scaffolding of mathematical language (Delaney, 2001). It is important to show children the range of ways to represent fractions because some children only see fractions as part of a whole (Haylock, 2014). This is why I have chosen not to use pizzas or chocolate bars to develop Liam's fraction knowledge, as I want him to understand the key ideas of fractions (Haylock, 2014; Rowland, 2009).

If this was a whole class misconception, I would model this activity using giant numicon and then give children opportunities to explore converting improper fractions into mixed numbers themselves with numicon. This is important because active resources allow children to develop relational understanding (Skemp, 1976; Delaney, 2001). Additionally, teachers can then facilitate learning through questioning whilst children manipulaute resources, enabling meaningful learning to take place (Delaney, 2010; Rowland, 2009). However, children need to be given time to explore their resources and therefore their understanding (Drews, 2007) which further provides opportunities for teachers to identify misconceptions, through listening to their mathematical reasoning (Barnett, 2012).

Because Pupils need opportunities to understand fractions as numbers (DfE, 2014; Haylock, 2014) I would use the Prowise ITP where we could count up and down in fractions, showing that fractions can be more than one and consolidating learning of mixed numbers (figure 8). This practise provides opportunities to develop fluency and consolidate learning, which further provides opportunities to identify and challenge misconceptions (Cockburn, 2008). This enactive resource consolidates the understanding that there are different interpretations of fractions, that children should be familiar with (Hansen, 2014). Further reinforcing that fractions can be greater than 1 and that fractions are a number on their own (Haylock, 2014; Rowland, 2009). I have chosen to use the Prowise ITP in particular because it contains a wide range of accurate fractions and clearly shows mixed numbers, which allows the task to be interactive and LTHC depending on the child's ability. Additionally, you could use a blank counting stick to count fractions, however I would avoid using playdough because it will be difficult to illustrate fractions accurately, possibly confusing children, as mixed numbers need to be explicitly modelled accurately.

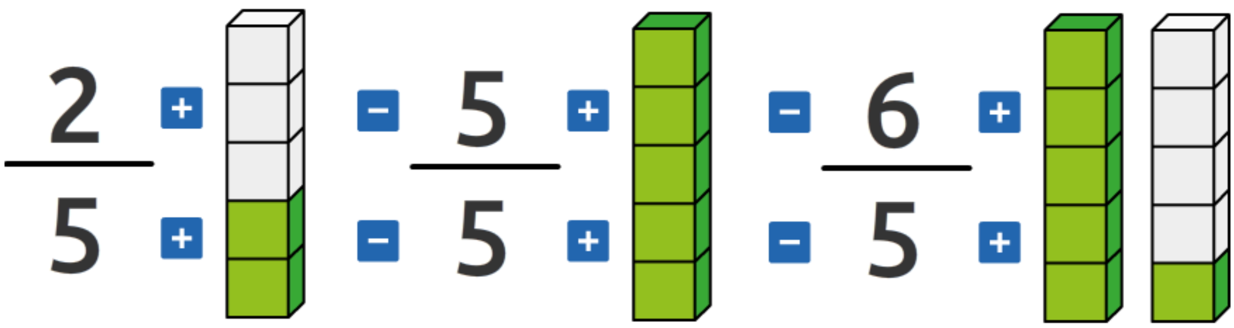


Figure 8

To assess relational understanding (Skemp, 1976) of mixed numbers I would then use a concept cartoon as illustrated in figure 9. This develops a classroom culture that supports identifying misconceptions in order to challenge them, rather than being scared to make mistakes (Hansen, 2014). Further providing opportunities for reasoning and meaningful talk between students as they discuss which character has the correct belief. Furthermore, through identifying the other characters misconceptions, children have further opportunities to explain their understanding through reasoning (Hansen, 2014; Cockburn, 2008). This also provides opportunities for progression as you could ask children to use their decimal and percentages understanding to check the order on the number line. With Liam's new relational understanding of improper fractions, he should be able to complete this task successfully, which will further consolidate his understanding. I can then use this as summative assessment, which will inform my future planning and therefore progression as I may need to teach children how to convert into equivalent fractions (DfE, 2014).

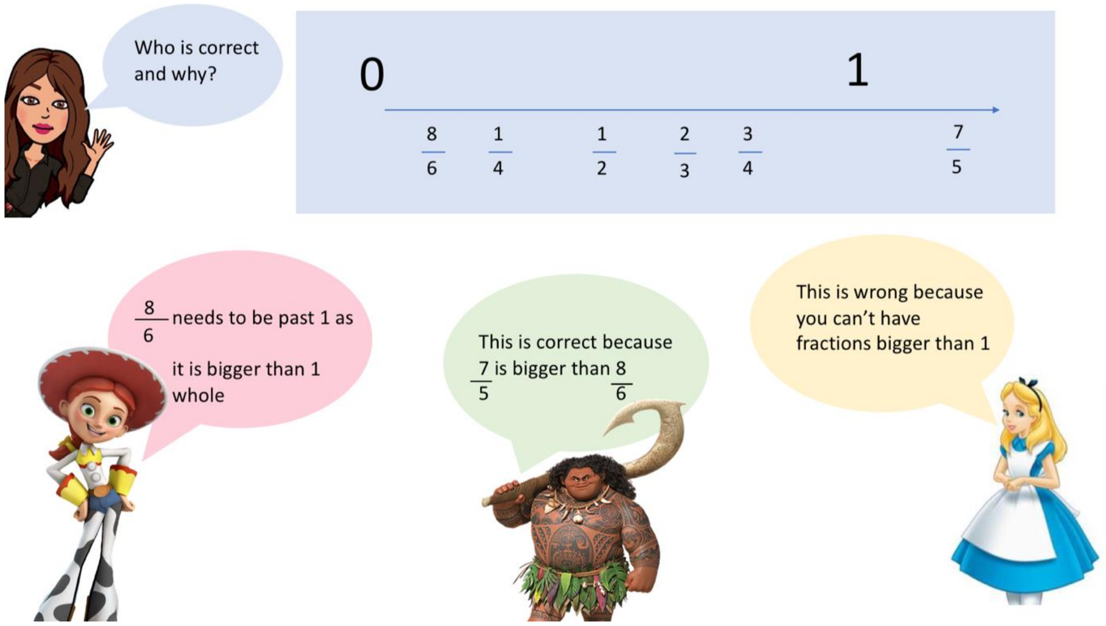


Figure 9

Drews (2007) argues that children need to be given time to reflect on their learning to be able to further develop their understanding (Ryan, 2007), which can be supported through diagnostic teaching (Ryan and Williams, 2010). This allows teachers to identify areas of weaknesses that need to be addressed to support relational understanding and then future learning (Barret, 2012). Which is why opportunities to explore mathematical concepts with resources, accompanied with teachers facilitating higher learning through critical questioning is detrimental to quality maths teaching (Delaney,2001).

Word count:2199

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